Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

FURTHER MATHEMATICS **9231/12**

Paper 1 Further Pure Mathematics 1 **October/November 2024**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 The sequence $u_1, u_2, u_3, \ldots$ is such that $u_1 = 4$ and $u_{n+1} = 3u_n - 2$ for $n \geqslant 1$.

Prove by induction that $u_n = 3^n + 1$ for all positive integers n. [5]

2 The line l_1 has equation $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} - 4\mathbf{k})$.

The plane Π contains l_1 and is parallel to the vector $2\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$.

(a) Find the equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

The line l_2 is parallel to the vector $5\mathbf{i}-5\mathbf{j}-2\mathbf{k}$.

(b) Find the acute angle between l_2 and Π. [3]

3 It is given that

$$\alpha + \beta + \gamma + \delta = 2,$$
$$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 3,$$
$$\alpha^3 + \beta^3 + \gamma^3 + \delta^3 = 4.$$

(a) Find the value of $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$. [2]

(b) Find the value of $\alpha^2\beta + \alpha^2\gamma + \alpha^2\delta + \beta^2\alpha + \beta^2\gamma + \beta^2\delta + \gamma^2\alpha + \gamma^2\beta + \gamma^2\delta + \delta^2\alpha + \delta^2\beta + \delta^2\gamma$. [3]

(c) It is given that α, β, γ, δ are the roots of the equation

$$6x^4 - 12x^3 + 3x^2 + 2x + 6 = 0.$$

(i) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [3]

(ii) Find the value of $\alpha^5 + \beta^5 + \gamma^5 + \delta^5$. [2]

4 The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 5 \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} 0 & -2 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} -2 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix}.$$

(a) Show that $\mathbf{CAB} = \begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix}$. [3]

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(b) Find the equations of the invariant lines, through the origin, of the transformation in the x-y plane represented by **CAB**. [5]

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Let $\mathbf{M} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$.

(c) Give full details of the transformation represented by **M**. [2]

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(d) Find the matrix **N** such that $\mathbf{NM} = \mathbf{CAB}$. [3]

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5 It is given that $S_n = \sum_{r=1}^{n} u_r$, where $u_r = x^{f(r)} - x^{f(r+1)}$ and $x > 0$.

(a) Find S_n in terms of n, x and the function f. [2]

(b) Given that $f(r) = \ln r$, find the set of values of x for which the infinite series

$$u_1 + u_2 + u_3 + \ldots$$

is convergent and give the sum to infinity when this exists. [3]

(c) Given instead that $f(r) = 2\log_x r$ where $x \neq 1$, use standard results from the List of formulae (MF19) to find $\sum_{n=1}^{N} S_n$ in terms of N. Fully factorise your answer. [4]

6 The curve C has equation $y = \dfrac{x^2+3}{x^2+1}$.

(a) Show that C has no vertical asymptotes and state the equation of the horizontal asymptote. [2]

(b) Show that $1 < y \leqslant 3$ for all real values of x. [4]

(c) Find the coordinates of any stationary points on C. [2]

(d) Sketch C, stating the coordinates of any intersections with the axes and labelling the asymptote. [3]

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(e) Sketch the curve with equation $y = \frac{x^2+1}{x^2+3}$ and find the set of values of x for which $\frac{x^2+1}{x^2+3} < \frac{1}{2}$. [4]

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7 The curve C_1 has polar equation $r = a(\cos\theta + \sin\theta)$ for $-\frac{1}{4}\pi \leqslant \theta \leqslant \frac{3}{4}\pi$, where a is a positive constant.

(a) Find a Cartesian equation for C_1 and show that it represents a circle, stating its radius and the Cartesian coordinates of its centre. [4]

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(b) Sketch C_1 and state the greatest distance of a point on C_1 from the pole. [3]

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The curve C_2 with polar equation $r = a\theta$ intersects C_1 at the pole and the point with polar coordinates $(a\phi, \phi)$.

(c) Verify that $1.25 < \phi < 1.26$. [2]

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(d) Show that the area of the smaller region enclosed by C_1 and C_2 is equal to

$$\tfrac{1}{2}a^2\left(\tfrac{3}{4}\pi + \tfrac{1}{3}\phi^3 - \phi + \tfrac{1}{2}\cos 2\phi\right)$$

and deduce, in terms of a and ϕ, the area of the larger region enclosed by C_1 and C_2. [7]

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Additional page

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