



# Cambridge International AS & A Level

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## FURTHER MATHEMATICS

9231/12

Paper 1 Further Pure Mathematics 1

October/November 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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- 1** The sequence  $u_1, u_2, u_3, \dots$  is such that  $u_1 = 4$  and  $u_{n+1} = 3u_n - 2$  for  $n \geq 1$ .

Prove by induction that  $u_n = 3^n + 1$  for all positive integers  $n$ . [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



**2** The line  $l_1$  has equation  $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} - 4\mathbf{k})$ .

The plane  $\Pi$  contains  $l_1$  and is parallel to the vector  $2\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$ .

(a) Find the equation of  $\Pi$ , giving your answer in the form  $ax + by + cz = d$ . [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



5

The line  $l_2$  is parallel to the vector  $5\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$ .

(b) Find the acute angle between  $l_2$  and  $\Pi$ .

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



$$\alpha^3 + \beta^3 + \gamma^3 + \delta^3 = 4.$$

[illegible]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, which is otherwise blank.



(c) It is given that  $\alpha, \beta, \gamma, \delta$  are the roots of the equation

$$6x^4 - 12x^3 + 3x^2 + 2x + 6 = 0.$$

(i) Find the value of  $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$ . [3]

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(ii) Find the value of  $\alpha^5 + \beta^5 + \gamma^5 + \delta^5$ . [2]

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$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 & -2 \\ -1 & 3 \\ 0 & 0 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} -2 & -1 & 1 \\ 1 & 1 & 3 \end{pmatrix}.$$

(a) Show that  $\mathbf{CAB} = \begin{pmatrix} 3 & -7 \\ -9 & 3 \end{pmatrix}$ . [3]

[illegible]

(b) Find the equations of the invariant lines, through the origin, of the transformation in the  $x$ - $y$  plane represented by **CAB**. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.





Let  $\mathbf{M} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ .

- (c) Give full details of the transformation represented by  $\mathbf{M}$ . [2]

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- (d) Find the matrix  $\mathbf{N}$  such that  $\mathbf{NM} = \mathbf{CAB}$ . [3]

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5 It is given that  $S_n = \sum_{r=1}^n u_r$ , where  $u_r = x^{f(r)} - x^{f(r+1)}$  and  $x > 0$ .

(a) Find  $S_n$  in terms of  $n$ ,  $x$  and the function  $f$ . [2]

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(b) Given that  $f(r) = \ln r$ , find the set of values of  $x$  for which the infinite series

$$u_1 + u_2 + u_3 + \dots$$

is convergent and give the sum to infinity when this exists. [3]

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- (c) Given instead that  $f(r) = 2 \log_x r$  where  $x \neq 1$ , use standard results from the List of formulae (MF19) to find  $\sum_{n=1}^N S_n$  in terms of  $N$ . Fully factorise your answer. [4]

[4]





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- (d) Sketch  $C$ , stating the coordinates of any intersections with the axes and labelling the asymptote. [3]

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- (e) Sketch the curve with equation  $y = \frac{x^2 + 1}{x^2 + 3}$  and find the set of values of  $x$  for which  $\frac{x^2 + 1}{x^2 + 3} < \frac{1}{2}$ . [4]

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The curve  $C_2$  with polar equation  $r = a\theta$  intersects  $C_1$  at the pole and the point with polar coordinates  $(a\phi, \phi)$ .

- (c) Verify that  $1.25 < \phi < 1.26$ . [2]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

- (d) Show that the area of the smaller region enclosed by  $C_1$  and  $C_2$  is equal to

$$\frac{1}{2}a^2\left(\frac{3}{4}\pi + \frac{1}{3}\phi^3 - \phi + \frac{1}{2}\cos 2\phi\right)$$

and deduce, in terms of  $a$  and  $\phi$ , the area of the larger region enclosed by  $C_1$  and  $C_2$ . [7]

[illegible]



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## Additional page

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